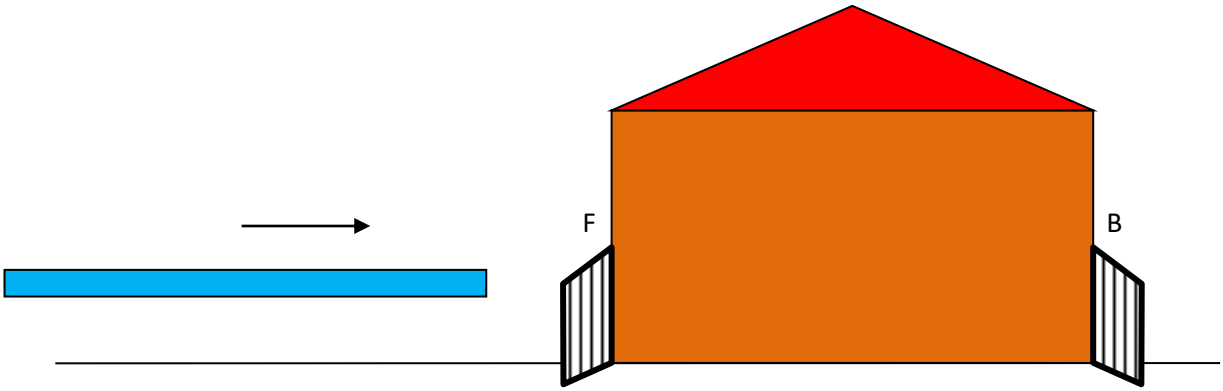
Teacher notes
Topic A

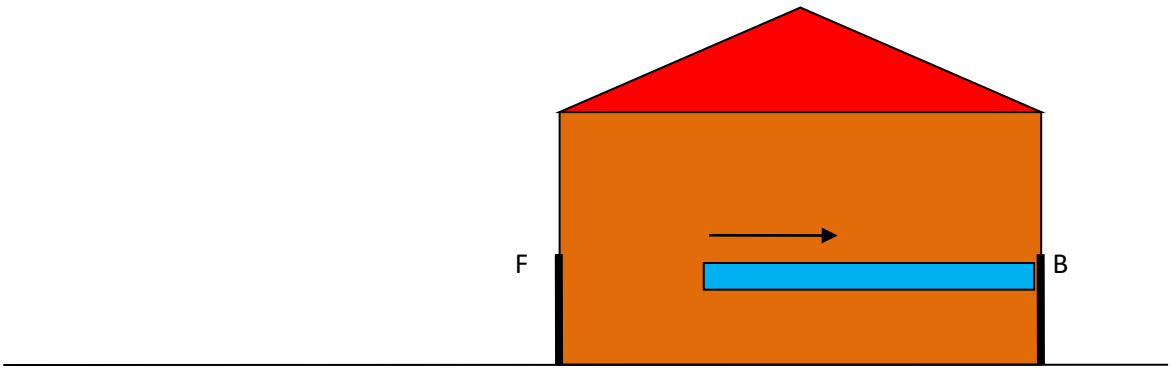
The pole in-the-barn "paradox": an exercise in simultaneity.

A pole and a barn have the same proper length. The pole moves at high speed towards the barn. At some point the doors of the barn will close (at the same time in the frame of the barn) and then immediately open again.



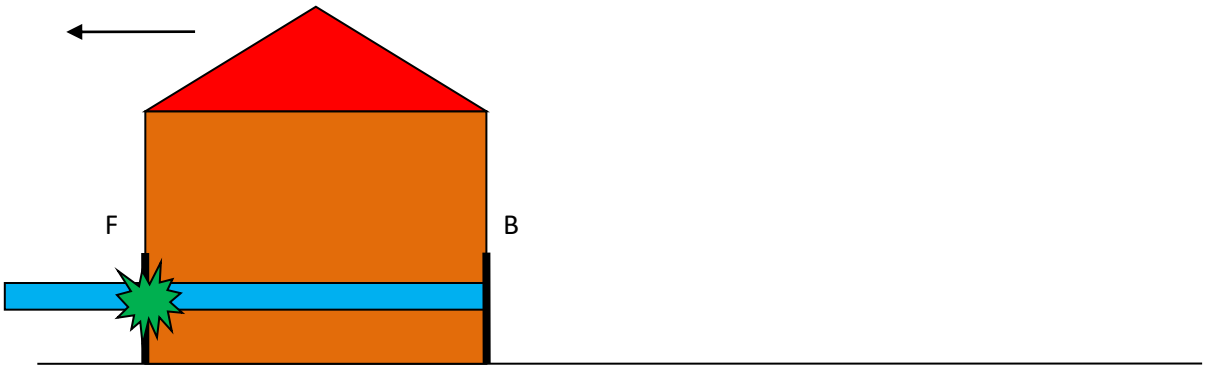
It can be argued that:

Viewpoint 1: Relative to the barn, the length of the pole will be Lorentz contracted and so the pole can fit within the barn. In other words, when the front and back doors of the barn **close at the same time** (in the barn's frame), the pole will be inside the barn. The diagram shows the case where the doors close (and immediately open) when the front of the pole has just reached the back door.



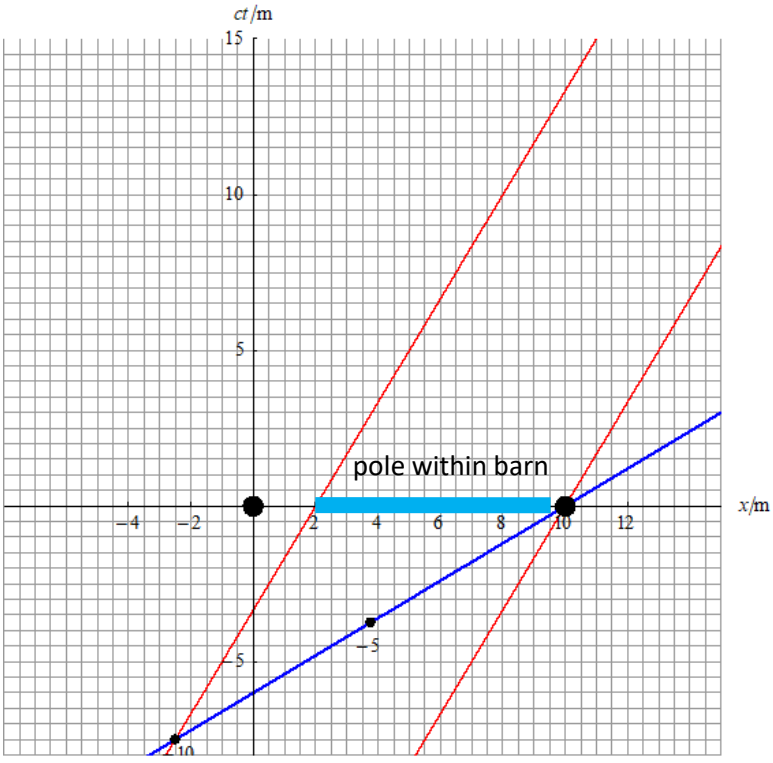
BUT, it can also be argued that:

Viewpoint 2: Relative to the pole, the barn is moving to the left, so its length is contracted. Hence the pole cannot fit within the barn. When the back door closes, part of the pole is outside the barn and the front door has smashed into the pole:



The two points of view are contradictory. Which viewpoint is correct?

The first viewpoint is shown on the spacetime diagram where the proper length of the rod and the barn are taken to be 10 m and the speed of the pole is 0.60 c. Let us call S the barn frame and S′ the pole rest frame. The origin of S′ is at x = 10 m, t =0.



The red lines are the worldlines of the left and right ends of the pole. The large circles indicate when the doors are closed (t = 0). The pole length is contracted to $\frac{10}{1.25} = 8.0\text{ m}$ and extends from x = 2 m to x =10 m in the barn frame. This is shown on the diagram. We see that the pole is within the barn at the instant both doors are closed. Viewpoint 1 looks correct. What about viewpoint 2?

It is clear from the spacetime diagram that in the pole frame the front door closes **after** the back door. S is the barn frame and S' is the pole rest frame:

$$\begin{aligned} c\Delta t' &= c(t'_{\text{B}} - t'_{\text{F}}) \\ &= \gamma(c\Delta t - \frac{v}{c}\Delta x) \\ &= \gamma(0 - \frac{v}{c}\Delta x) \\ &= -\frac{5}{4}\times 0.60 \times 10 \\ &= -7.5\text{ m} \end{aligned}$$

In the pole frame the front door closes $c\Delta t' = 7.5\text{ m} \Rightarrow \Delta t' = 2.5\times 10^{-8}\text{ s}$ **after** the back door.

The spacetime diagram above assumes that the back door closes at t = 0 just as the front of the pole arrives at the back door. (The door will immediately open again so the pole will not crash into the door.) The barn length is contracted to $\frac{10}{1.25} = 8.0\text{ m}$ so 2.0 m of the pole is outside the barn.

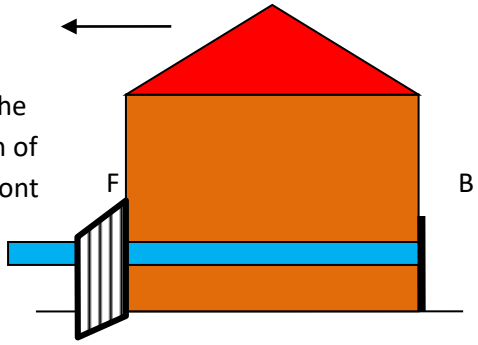
In the pole frame, the front door will close after $\Delta t' = 2.5\times 10^{-8}\text{ s}$. In this time the barn moved a distance $0.60c\Delta t' = 4.5\text{ m}$ to the left.

This means the right end of the pole is 4.5 m out of the barn and a length 5.5 m of the pole is within the barn.

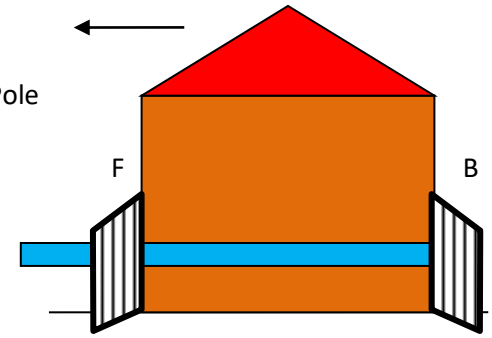
These results mean that the pole goes through the barn without smashing into any of the doors.
This is consistent with the events of viewpoint 1 and there is no longer a contradiction.

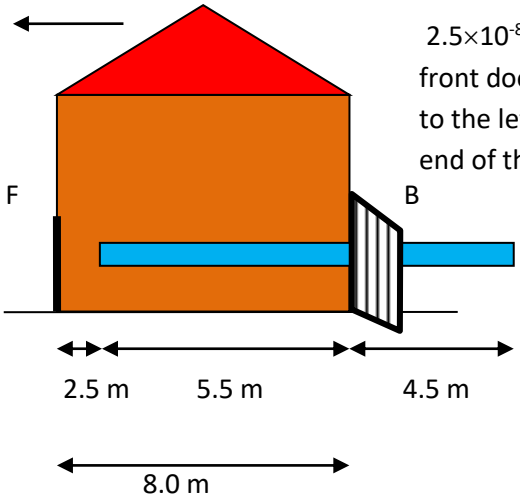
These events are shown in the figures below.

Back door closes as the front of the pole arrives there. A length of 2m of the pole is outside the barn. Front door is open.



Back door immediately opens. Pole does not touch the door.





2.5×10^{-8} s later in the pole frame, the front door closes. The barn has moved to the left a distance 4.5 m. The left end of the pole is within the barn.